Measuring long run marginal cost for pricing

Dr Richard Tooth

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<td>Unit 3, 97 Northbourne Ave</td>
<td>Level 2, 65 Southbank Boulevard</td>
</tr>
<tr>
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<td>PO Box 2475</td>
<td>GPO Box 220</td>
<td>Turner ACT 2612</td>
<td>GPO Box 3179</td>
</tr>
<tr>
<td>Wellington 6140</td>
<td>Auckland 1140</td>
<td>NSW 2001</td>
<td>GPO Box 252</td>
<td>Melbourne, VIC 3001</td>
</tr>
<tr>
<td>Ph: +64 4 915 7590</td>
<td>Ph: +64 9 913 6240</td>
<td>Ph: +61 2 9234 0200</td>
<td>Ph: +61 2 6267 2700</td>
<td>Ph: + 61 3 9626 4333</td>
</tr>
<tr>
<td>Fax: +64 4 915 7596</td>
<td>Fax: +64 9 913 6241</td>
<td>Fax: +61 2 9234 0201</td>
<td>Fax: +61 2 6267 2710</td>
<td>Fax: + 61 3 9626 4231</td>
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For information on this report please contact:

Name: Dr Richard Tooth
Telephone: 02 9234 0216
Mobile: 0412 105 817
Email: rtooth@srgexpert.com
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# Glossary

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<td>AIC</td>
<td>Average Incremental Cost</td>
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<td>Commission</td>
<td>Essential Services Commission of South Australia</td>
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<tr>
<td>Estimation Period</td>
<td>The period over which LRMC is estimated; often a period of many (e.g. 25) years</td>
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<td>LRIC</td>
<td>Long Run Incremental Cost</td>
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<td>LRMC</td>
<td>Long run marginal cost</td>
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<td>LRMOCC</td>
<td>Long run marginal operational cost</td>
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<td>LRMCC</td>
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<td>PV</td>
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Summary

Introduction

A commonly recognised starting point for efficient utility pricing is the setting of usage prices to marginal cost (the cost of supplying an additional unit). However, there are debates as to how marginal cost should be measured.

This paper examines the economic issues of long-run marginal cost (LRMC) pricing. It is a companion paper to three other papers by Sapere Research Group (Sapere) prepared for the Essential Services Commission of South Australia (Commission) with regards to its inquiry into LRMC pricing of the water and wastewater services provided by SA Water.

Overview of LRMC

LRMC is commonly defined as the cost of supplying an additional unit assuming that all factors of production can be varied. However, in practice all factors cannot be varied and LRMC is estimated from forecasts of the costs of meeting a future increment in demand over a long period.

LRMC is most commonly estimated using one of two methods, which using the common terminologies applied in the water industry, are:

- the Turvey perturbation (Turvey) method, which involves estimating the present value (PV) of the change in future costs required due to a marginal but permanent change in forecast demand
- the average incremental cost (AIC) method, which involves estimating the PV of future costs required due to the forecast demand that is in excess of current demand.

The methods are very similar; both are measures of the PV of costs incurred in meeting additional demand averaged over a time period and unitised to give a per-unit measure of cost. The methods differ primarily in the weight they give to costs in different time periods. The AIC method places greater weight on costs in the near future.

Issues and implications of LRMC pricing

Pricing based on LRMC has been advocated to address issues with pricing based on short-run marginal cost (SRMC) SRMC is also a measure of marginal cost, but differs to LRMC in assuming that at least one factor of production (generally capital) cannot be varied.

Pricing at SRMC can lead to substantial price variation when expanding capacity involves ‘lumpy’ (large indivisible) investments. When meeting increasing demand involves lumpy investments, a SRMC price will follow a saw-tooth pattern whereby the price is low when there is excess capacity, increases as capacity is constrained, and then fall suddenly once additional capacity is installed.

The price variability caused by lumpy investments has a number of undesirable consequences:
1. It may be in itself undesirable to the extent that consumers and suppliers prefer price stability.
2. It causes a financing issue, as the price fall following an augmentation limits the ability of investors to recover the cost of their investment.

3. It can dilute the effectiveness of the price signal if consumers, and potentially suppliers, expect prices to continue along a stable path.

Pricing at LRMC helps to address these issues. In effect, LRMC pricing is a method of explicitly including the capital costs of future augmentations and stabilising prices over time.

However, there are also issues with pricing at LMRC. Pricing at LRMC dampens but does not remove the saw-tooth pricing pattern. A more common concern is that pricing at LRMC can send an inefficient price signal in the short-term; for example, during periods of excess capacity, LRMC will be greater than SRMC and pricing at LRMC may discourage efficient use. Furthermore, as LRMC is based on future supply and demand, LRMC estimates can be sensitive to the time period selected and demand (and supply) assumptions.

**Pricing with regard to LRMC**

Pricing with regard to LRMC requires balancing the costs and benefits of alternative choices. The key choices relate to how prices should vary over time and how prices should be modified to reflect uncertainty. The more that costs and demand vary and are uncertain, the more important these choices will be.

For the purpose of providing a signal to consumers and suppliers for efficient use, the preferred choices will depend on responsiveness of consumers and suppliers to price signals. For example, the more responsive they are in the short-run the greater the case for SRMC pricing. The responsiveness to price will depend on a range of factors including the magnitude, direction and speed of price changes and how prices are communicated. Such factors may vary by industry. There is a case for generally stabilised prices with adjustments (temporary or permanent as necessary) for shocks that would raise prices (e.g. lower supply or rapid increases in demand).

The equity implications of price changes are also important. Usage prices set with regard to LRMC may change significantly over time. To achieve cost recovery a change in usage pricing will require changes to other charges (e.g. connection charges). It is appropriate to have a clear framework for how these changes will be implemented.
1. Introduction

Marginal cost is defined as the cost of supplying an additional unit of good or service.

The concept of marginal cost is critical for pricing of utility services. A commonly recognised starting point for efficient utility pricing is the setting of usage prices to marginal cost. As stated by Alfred Kahn:²

_The central policy prescription of microeconomics is the equation of price and marginal cost. If economic theory is to have any relevance to public utility pricing, that is the point at which the inquiry must begin._

Pricing can provide a signal to both consumers and suppliers for efficient use of resources. A price set below marginal cost can encourage an individual to consume additional units even when the benefits to the individual are outweighed by the costs to society. Conversely a price set above marginal cost can discourage individuals from consuming additional units despite the benefits to them outweighing the costs to society.

Despite its attraction, marginal cost pricing is not straightforward to apply. A common challenge in utility pricing occurs when large investments are required to expand output. In such situations, economists often recommend that pricing be set with regards to long run marginal cost (LRMC).³

This paper examines the economic issues of LRMC pricing. It is a companion paper to three other papers by Sapere Research Group (Sapere) prepared for the Essential Services Commission of South Australia (Commission) with regards to its inquiry into pricing of water and wastewater services provided by SA Water.⁴ While the paper has been developed in the context of a review of water and wastewater service pricing, the principles discussed are largely common to most utilities.

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¹ For the purposes of public utility pricing, the costs considered should include all societal costs (e.g. including environmental externalities).
² As recorded in Kahn (1988), page 65.
³ The National Water Initiative pricing principles, which have been agreed to by state and territory governments, include the principle that drinking water prices shall be set with regard to LRMC.
⁴ These other papers related to LRMC pricing of the drinking water, sewerage and trade waste services that are provided by SA Water.
2. The economics of LRMC pricing

2.1 LRMC in theory

The standard definition of long-run marginal cost (LRMC) is the cost of supplying an additional unit (the marginal cost) assuming that all factors of production can be varied. In contrast, short run marginal cost (SRMC) is the cost of supplying an additional unit assuming that at least one factor of production (hereafter in this report assumed to be capital investment) is fixed.

Like marginal cost, both SRMC and LRMC are forwarding looking concepts; that is, costs that are sunk (i.e. that have been incurred or are committed to be incurred) are irrelevant in their calculation.

In describing the distinction between short run and long run costs it useful to start by examining average costs (defined as total costs divided by total output). Figure 1 provides an illustrative example of average costs. It shows, in the short and long run, average costs for different levels of output. The short run average cost (SRAC) curves depend on the level of the capacity investment. As shown in the figure SRAC follows a U-shaped pattern first decreasing (often due to averaging of fixed costs over greater output) and then increasing (often due to capacity constraints).

The long run average cost (LRAC) curve is the minimum value of the SRAC cost curves. Thus, if the amount of the fixed factor (e.g. capital) is optimal then SRAC will equal LRAC.

Figure 1: Average cost curves
The relationship between marginal cost and average cost is also shown in the figure. When average cost is decreasing the marginal cost must be less than average cost and vice-versa\(^5\) and at the minimum values of average cost, marginal cost will equal average cost. This rule applies to short and long run costs; SRMC will equal SRAC at the SRAC minimum values. Similarly LRMC will equal LRAC when LRAC is at its minimum.

Since LRAC is the minimum values of SRAC, when LRAC is at its minimum SRAC=SRMC=LRMC=LRAC. Figure 1 illustrates a situation whereby there are discrete levels of capacity\(^6\) and therefore LRAC may increase over short ranges of demand, reach a cusp (at which a higher level of capacity is optimal) and then decrease. If the level of capacity was infinitely divisible then the LRMC would simply represent the minimum value of the SRAC curves.

If each new investment could be sized to any capacity and made at the same average cost then the LRAC curve would be flat and LRMC would be the same as LRAC. This may be a reasonable assumption for many industries where location is of little importance; however for public utilities such as water, LRAC will (above a minimum efficient scale) tend to increase with greater supply.\(^7\) This is due to resource constraints at a local geographic level with the implication that each new augmentation is at a higher cost than the previous augmentation.

### 2.2 LRMC in practice - overview

In practice estimates of LRMC vary from the textbook definition of LRMC. The textbook definition of LRMC includes the assumption that all factors of production (including capital investments) can be varied. But in practice, this assumption is unrealistic; facilities such as a desalination plant are built to manage a range of demand levels and therefore will rarely be of the optimal size for a particular level of demand.

To deal with this limitation, the approach used in practice (described in the next sub-section) is to estimate LRMC by examining projections of demand and costs over a long time into the future. However, because demand changes over time, investment decisions need to reflect future as well as current demand and the capital investments will rarely be optimal.

There are number of important implications of this practical approach to estimating LRMC.

1. Estimating LRMC in practice involves a time dimension; specifically it involves estimating costs (and demand) over future periods and converting these to present day values. In contrast SRMC (as in theory) is measured over a single period. In practice, the key distinction between SRMC and LRMC is the time period over which measurement is made.

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\(^5\) For example, if the average cost of producing a unit is $1 per unit and the marginal cost is $2 per unit, then the production of additional units will increase the average cost and vice-versa.

\(^6\) This may be because, for example: the number of lanes on a bridge must be whole numbers; or, the capacity of desalination plants may be limited to particular sizes.

\(^7\) Industries may be classified as increasing cost, constant cost, and decreasing cost industries depending on how LRAC changes with demand. LRAC may decrease for some industries due, for example, to increase economies of scale in factors of production.
2. The relationship between SRMC and LRMC differs between practice and theory. When the capital program is optimal, then in theory SRMC will equal LRMC (as illustrated in Figure 1 above) but in practice because they are estimated over different time periods they can differ — SRMC may be above or below LRMC.8

3. The practical definition of LRMC differs from marginal cost and, therefore, a price set at LRMC may encourage inefficient use of a resource.

2.3 Methods of estimating LRMC

2.3.1 Introduction and overview

There are a number of methods used to estimating LRMC.9 Unfortunately there are inconsistencies in the literature in the terminology used. The discussion below reflects (what appears to be) the most common terminology. The literature is also sometimes vague and/or inconsistent in describing how the methods should be applied. The description applied below is that most consistent with the goal of sending a price signal to consumers and producers for efficient decisions over the long-run. For example, an initiative that reduces demand over the long-run at a lower cost than the LRMC should reduce total costs.

The terminology that is commonly used varies by industry. The two most common methods for estimating LRMC in the water industry and discussed in more detail below are:

- the Turvey perturbation (Turvey) method, and
- the Average Incremental Cost (AIC) method.

In other industries (e.g. aviation and telecommunications) it is common to refer to Long Run Incremental Cost (LRIC) and its variants.10 LRIC is commonly defined as the ‘incremental costs that arise in the long-run with a specific increment in the volume of production’.11 The Turvey and AIC methods may be thought of per-unit measures of the LRIC using different size increments.

The Turvey and AIC methods share a similar approach. Both methods involve forecasting costs and demand over a long time period (the Estimation Period) and estimating LRMC as the present value (PV) of costs required to meet a change in future demand divided by the discounted sum of the change in future demand. For summation, demand is discounted using a PV formula12 and, consistent with the literature, this paper refers to the PV of

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8 This is because in practice LRMC is based on future demand and costs.
9 There are also other methods that are used to calculate prices (e.g. simple average cost pricing) that are not forward looking and therefore out of scope of this project.
10 The variants include total service long-run incremental cost (TSLRIC) and Total element long-run incremental cost (or TELRIC).
12 The PV of demand is in the units of demand (e.g. kilo-litres). The approach of discounting future demand like financial values can be justified on the basis that, if price is set to LRMC, the PV of revenue obtained from the change in future demand will equal the PV of the costs required to meet the change in future demand.
demand to refer to the sum of demand that has been discounted using a PV formula. Both methods may be thought of as providing a time-averaged estimate of marginal costs.\textsuperscript{13}

For ease of calculation, LRMC is typically calculated using real values (that is adjusted to remove effects of general price changes). The comments and formulas presented in this paper reflect a real-value approach.

It is also often useful (for ease of calculation and transparency) to employ a 'building block approach', whereby the LRMC is estimated as the sum of different components (i.e. the building blocks) which are separately estimated. For example, it is often useful to estimate costs LRMC as the sum of long run marginal operational cost (LRMOC) and long-run marginal capital cost (LRMCC). The LRMOC component is generally thought to be relatively simple to calculate as it is largely insensitive to the method used.

2.3.2 The Turvey perturbation method

The Turvey perturbation method is based on approach developed by Ralph Turvey (1970) of measuring LRMC by considering how changes in demand affected the timing of investments. The method has been referred by a number of different names including the perturbation method/approach, the Turvey method/approach (e.g. Mann 1993), Long Run Incremental Cost (LRIC) (London Economics, 2001), LRMC (Vass 2003) and Marginal Incremental Cost (MIC) (Marsden Jacob Associates 2004).\textsuperscript{14} In this and the related papers for the Commission it is referred to as the Turvey LRMC.

It is calculated as:\textsuperscript{15}

\[
\text{Turvey LRMC} = \frac{PV \text{ of change in costs due to a permanent change in demand}}{PV \text{ of permanent change in demand}}
\]

The Turvey method is based on estimating the effect on costs of making a marginal but permanent change in demand. It involves:

1. forecasting demand over the Estimation Period
2. selecting the optimal (i.e. least cost) capital program to meet that demand
3. modifying the demand forecast by a hypothetical small permanent adjustment
4. estimating the new optimal capital program, and
5. estimating LRMC as the difference in the PV of the costs in meeting demand (i.e. PV of the costs associated with 4. less 2.) divided by the PV change in demand.

The method requires selecting a permanent change in demand. This may be a permanent increase (e.g. the green portion illustrated in Figure 2 below) or a permanent decrease. A

\textsuperscript{13} In contrast estimates of SRMC are typically based on current variable costs. Note, sometimes LRMC is described as being an adjustment to SRMC to allow for capital costs. This is not correct. SRMC may exceed the estimates of LRMC due to how the methods are estimated.

\textsuperscript{14} The Turvey approach appears to be the most common method used in Australia, although there are some notable exceptions including SA Water's use of the AIC method.

\textsuperscript{15} This is equivalent to dividing the annuitized costs by the annual change in demand.
common concern in applying the Turvey method is that the results appear to be sensitive to the level of permanent demand chosen. This is generally because discrete (e.g. yearly) time steps are used for the analysis and a small difference in the permanent change can change the period (e.g. financial year) in which an investment is assumed to occur. To overcome this issue a pragmatic approach is to choose the permanent change in demand to match the growth rate in demand so the effect of the permanent change being tested is to bring forward (or push back) investment by a year.\textsuperscript{16} Alternatively adjustments might be made for smaller time steps.

\textbf{Figure 2: Future demand}

![Future demand diagram]

\subsection*{2.3.3 Average incremental cost}

The common alternative to the Turvey method is to apply the average incremental cost (AIC) method. This is calculated as:

\[ AIC = \frac{PV \text{ of future costs to meet incremental demand}}{PV \text{ of future incremental demand}} \]

Conceptually the AIC method is also simple to apply. It involves:

1. determining current demand
2. determining the ongoing costs to meet current demand
3. forecasting the future incremental demand; that is, forecast demand over the Estimation Period less current demand (by time period)

\textsuperscript{16} For example if demand is growing by 2 units a year, then it is convenient to use 2 units as the permanent change in demand.
4. determining the optimal (i.e. least cost) program to meet the increase in demand, and

5. estimating LRMC as the PV of the additional costs of meeting the future increase in demand (i.e. PV of the costs associated with 4. less 2.) divided by the PV of the future increase in demand (i.e. PV of 3. less 1.)

The future incremental demand in the denominator is the grey area in Figure 2. As reflected in the figure, the baseline from which the increment is measured is the current level of demand (not the current level of capacity) and the upper bound of the increment is the forecast demand (not the forecast level of capacity).

Care is required in measuring future costs. It is preferable to use annuitized values of the capital, or, as put by Mann and Beecher (1996, page 3) ‘the annual payment over the useful service life of the capital expenditure necessary to pay financing costs and fully recover the additional capacity costs.’ A cash flow approach may be used if allowance (a deduction on the costs) is made for the residual value of the assets post the planning period so as to ensure that the costs measured are consistent with the demand.

2.3.4 Comparing the Turvey and AIC methods

Overview
The Turvey and AIC methods both adopt a similar approach. Both are, in effect, a measure of the costs due to a change in demand discounted and averaged over a time period. They differ in the measure of demand they consider:

• The Turvey method is based on an increment (or decrement) to forecast demand;
• The AIC method is based on an increment to current demand.

How the results of the methods differ
In a number of circumstances the AIC and Turvey methods will produce identical results, however there are sources of differences.

While some reports have found large differences in results from the two methods, these have generally been a result of an incorrect application of the formulas. For example, AIC is sometimes calculated to be significantly higher than the Turvey LRMC because (incorrectly) a cash flow basis is used with no allowance made for the residual value of assets. In some cases the Turvey LRMC is found to be high due to the use of the wrong denominator (in some cases a single period’s demand is used).

An illustration of the potential similarity in the results can be demonstrated using some simple algebra (see Box 1). In the example, both methods are shown to be a weighted average of the same sets of costs; the key difference in the two methods is in the weighting the two methods give to costs of meeting additional demand as demand changes over time. In effect, with respect to the (discounted) costs of meeting additional demand:

• the Turvey method gives equal weight to the marginal cost of meeting additional demand over the planning period.
• the AIC method gives greater weight to the marginal costs of additional demand in the near term.
Therefore, for example, if the marginal cost of meeting additional demand is increasing over time then the Turvey method will give a higher result than the AIC method. If however the marginal costs are relatively stable then the two methods will give similar results.

There appears to be the general view that the Turvey method is the more accurate approach to estimating LRMC and that the AIC method is a convenient approximation. A possible argument for the AIC approach is that costs in the near term are more likely to be known with greater certainty and thus should be given greater weight. However, uncertainty might be more explicitly addressed via other methods.

Box 1: Simple algebraic comparison of AIC and Turvey methods

To compare the different methods it is useful to use a simple algebraic model. Assume that we wish to estimate LRMC over a planning period from time $t = 0$ to $t = T$. Assume also that:

- demand is growing at a constant amount of $q$ units per year
- the discount rate is $r$
- supply is increased (at least cost) by investments with an annuitized cost of $ml_t$ (which includes allowance for depreciation/replacement), and
- per-unit operating costs are $c_t$.

For calculating Turvey LRMC consider a permanent reduction of $q$ units and therefore the numerator is reduction in costs achieved by delaying each investment by a single year. The Turvey LRMC is then simply:

$$\text{Turvey LRMC} = \frac{\sum_{t=0}^{T} d_t (ml_t + qc_t)}{\sum_{t=0}^{T} d_t q} = \sum_{t=0}^{T} d_t \left( \frac{ml_t}{q} + c_t \right)$$

Where $d_t$ as the discount factor for period $t$; that is $d_t = (1 + r)^{-t}$ and $\sum = \sum_{t=0}^{T}$

The structure of AIC LRMC is remarkably similar. Under the given assumptions the AIC LRMC simplifies to (see Appendix for details):

$$\text{AIC LRMC} = \frac{\sum_{t=0}^{T} w_t (ml_t + qc_t)}{\sum_{t=0}^{T} w_t q} = \sum_{t=0}^{T} w_t \left( \frac{ml_t}{q} + c_t \right)$$

Where $w_t = \sum_{j=t}^{T} d_j$; that is the sum of remaining discount factors to period $T$.

The AIC and Turvey methods may be considered weighted averages of the future annuitized costs. The AIC gives greater weight to earlier time periods and so the AIC method will give a smaller result when costs are increasing.

Source: Appendix 1.

For example (Mann and Beecher, 1996, page 3) state ‘the AIC approach offers the advantage of generating MCC estimations that approach theoretical long-run marginal cost and that can be directly used as the basis for commodity rates.’
**Practical differences**

There are also some practical differences in the two methods. In one respect the AIC method may seem simpler as cost information may be taken directly from forward capital and operational plans, whereas the Turvey method requires estimating costs under a second scenario. Nevertheless, both methods require clearly identifying expenditure that is growth related.

A practical advantage of the Turvey method is that it can be used to more easily estimate LRMC associated with costs that have multiple cost drivers. For example, when there are multiple drivers (e.g. quantity and quality drivers) for a new investment some method is required to isolate the impact of the driver of interest. By analysing the impact of a single driver, the Turvey method provides a means of clearly doing so.

**2.3.5 Other methods**

There are other cost measures/pricing rules that, consistent with LRMC principles, are forward looking and capture the cost of future investment.

Two other marginal cost measures of interest are described below. Other modifications to LRMC pricing rules are discussed in the Section 4 below.

**Annuitized unit cost of next supply source**

A possible measure of LRMC is the annuitized ‘unit cost’ of the next supply source, which is the annuitized capital and operating cost divided by the annual capacity provided by the supply source. Such an approach differs from the main measures of LRMC in that it is independent of the amount produced and does not vary over time. The annuitized unit cost may approximate the average of LRMC (as estimated by the Turvey or AIC methods) over a period of time. The annuitized unit cost is also relatively straightforward to calculate and therefore is a useful approximation for estimating LRMC.

**Marginal cost pricing with storage**

Water, in some situations (and potentially, in the future, energy), can be stored cheaply in large quantities. When adequate storage is available the distinction between SRMC and LRMC blurs and an alternative pricing rule may be appropriate.

When water is stored, the cost of using water includes the opportunity cost of not using that water in the future. Through this inter-temporal relationship, the SRMC of water will reflect (assuming adequate storage) the capital costs of augmentation in the future. In effect, storage can serve to ‘smooth over’ lumpy investments (discussed in the following section) blurring the distinction between the short and the long run. The key insight from this method is that, when water is stored, the efficient LRMC price changes at a different rate to that implied by

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18 See the appendix for a demonstration of this.
the Turvey and AIC methods. This method could be reasonably easily estimated as a modification to the AIC or Turvey methods.¹⁹

3. Issues and implications of pricing at LRMC

3.1 The rationale for LRMC

As noted in the previous section, LRMC, as estimated in practice, is not marginal cost. Why then is LRMC advocated?

The rationale for the departure from marginal cost lies in the problem of setting prices when future capital investments are lumpy; that is when investments are significant and indivisible.

When investments are lumpy, and the product cannot be cheaply stored, a SRMC price may be highly variable, following a saw-tooth pattern whereby prices are very low when there is excess capacity, very high when capacity is stretched and drop significantly once additional capacity comes on line.

This effect is explained using Figure 3 (on page 12) and illustrated in Figure 4 (on page 13) below. Figure 3 shows supply curves (which reflect SRMC) before and after a capacity augmentation and different demand curves representing increasing levels of demand.20 The intersection of these curves is the price that balances supply and demand. As demand increases the balancing price initially moves very little but then rapidly increases. Once a new augmentation is made, the supply curve shifts to the right and the balancing price drops sharply.

The price variability caused by lumpy investments has a number of undesirable consequences:21

First, price variability may be in itself undesirable to the extent that consumers and suppliers prefer price stability. There are a number of possible reasons for this including, for example:

- the administrative simplicity (for suppliers and consumers) of stable pricing,
- the simplicity of cost planning for large consumers,
- perceptions of fairness (e.g. over time), and
- consumer preferences for pricing stability.

Second, price variability causes a financing issue. Following augmentation the marginal-cost price falls, thereby limiting the ability of investors to recover the cost of their investment. An implication is that investors may wish to augment later than when it is socially optimal to do so.

20 The SRMC curve is unlikely to be vertical as capacity constraints are generally never absolute, but rather may be stretched at increasing cost. For example, the output of a production facility may be increased at higher short run costs through the use of overtime or more aggressive maintenance. As costs increase, other sources become viable. Similarly, the effective capacity of pipelines might be marginally increased through higher pumping costs or through storage to help alleviate peak demand constraints.

21 Another issue is the possibility of congestion costs associated with high demand. However such costs could be reflected in the SRMC.
so. While with utilities such as water, connection charges may be increased to achieve cost-
recovery, this may be difficult to achieve without having adverse social consequences as such
a pricing policy would result in large users benefiting most from the capacity increase but all
users being forced to share in the cost of the increases.

Third, the efficiency of the long run price signal may be diluted if consumers, and potentially
suppliers, expect prices to continue along a stable path. For example:

- Many consumers are likely to be unaware of the future price path. Generally, prices are
  set for regulatory period of a few years and how prices may change in the future is not
  made explicit.
- Even if consumers are aware of the future pricing path they may assume (perhaps with
  good reason) that governments would be sensitive to public opinion and prevent large
  increases in prices in the future.

Pricing at LRMC helps to address these issues. In effect, LRMC pricing is a method of
explicitly including the capital costs of future augmentations and smoothing the price over
time.\textsuperscript{22}

\textbf{Figure 3: Pricing and lumpy investments}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig3.png}
\end{figure}

\subsection*{3.2 Implications and issues of LRMC pricing}

While LRMC helps to address some issues, there are also some unwanted implications of
LRMC pricing.

\textsuperscript{22} Of note, if capacity augmentations were perfectly divisible (i.e. not 'lumpy') then the marginal cost would
include capital costs.
3.2.1 LRMC dampens but does not remove price variability

LRMC serves to dampen, but not remove, the variability in prices that is prominent with SRMC pricing. When recalculated over time, the LRMC values, whether estimated by the Turvey or AIC methods, will also follow a saw-tooth pattern as demand grows.

The pricing paths are illustrated in Figure 4 below. The black line shows how LRMC changes over time. As the time to augment approaches LRMC increases. This reflects that the cost (benefit) of a permanent increase (decrease) in usage increases (decreases) the closer a new augmentation is required. Once the capital expenditure is treated as a sunk cost, the value will drop. As shown in the figure the drop in the price is less significant than would occur if pricing were at SRMC (assuming no storage).

This change in values under LRMC pricing can be significant. Under common assumptions the capital component of the Turvey LRMC will escalate at the cost of capital. Using a 6.5% real cost of capital, this component would double every 11 years. Similarly if the next investment is expected to be 11 years into the future the capital component of the LRMC value would halve once the investment was sunk.

<table>
<thead>
<tr>
<th>Price/cost per unit</th>
<th>New capacity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;LRMC (Turvey) pricing&quot;</td>
<td>Drop in LRMC following new capacity</td>
<td>&quot;SRMC no storage&quot; pricing</td>
</tr>
</tbody>
</table>

Note: See appendix for a algebraic proof of how the Turvey LRMC increases over time.

Under marginal cost pricing with storage the rate of increase in LRMC is even more significant so as to reflect the marginal cost of storage (which for water may largely be the marginal rate of evaporation23).

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23 The marginal rate of evaporation is different to the average rate of evaporation and will depend on the shape of the storage (and other factors).
As discussed in the beginning of this section, there are further issues with a saw-tooth variability pattern in water prices. Perhaps the most significant issue relates to one of equity (i.e. fairness). If the saw-tooth pricing path is followed, then following an augmentation, fixed charges would need to rise to offset the fall in usage prices to achieve cost-recovery. An implication is that bills for small users may rise significantly even if their consumption hardly changes over time. In effect, the burden of the costs may be poorly aligned to the beneficiaries of an augmentation.

The existence of the saw-tooth pattern also creates some practical issues with the calculation of the LRMC values. These include that:

• the timing of when LRMC is measured can matter significantly; that is, the forward looking LRMC value should be much smaller once the augmentation is sunk, and
• if the AIC and Turvey methods were strictly followed, the LRMC values should be modified regularly over time to reflect the changes in values.

### 3.2.2 LRMC is a departure from marginal cost

A more common concern raised with pricing at LRMC is that because LRMC is a departure from marginal cost (which when there is excess capacity is SRMC) pricing at LRMC can distort decisions (i.e. discourage efficient or encourage inefficient use). When there is excess capacity, LRMC pricing is likely to be higher than SRMC and discourage efficient use; for example, a high LRMC water price may discourage consumers from watering their garden. Conversely when capacity is constrained LRMC may be below SRMC and LRMC pricing will encourage excessive use. This is illustrated in Figure 4 above: when there is excess capacity LRMC will be in higher than SRMC; at other times LRMC may be significantly below SRMC.

The significance of these distortions depends on the extent to which consumers and supplier can respond to short-term price signals. The extent to which this occurs will vary by industry and circumstance. Demand responsiveness to price will be less (less elastic) in the short term than in the long term; however, there is still significant evidence that consumers respond to short-term price signals. In the water industry there is strong evidence that consumers have responded to short-term increases in price — for example, Kenney et al. (2008) and Loaiciga and Renehan (1997) find evidence of consumers responding to short-term price increases during a drought.

### 3.2.3 Uncertainty and variation in supply and demand

Another challenge with pricing at LRMC is that the calculation of LRMC depends on future demand and supply, which are often uncertain. When future demand and supply is uncertain, a probabilistic model may be used to estimate an expected value; however, forecasts may vary significantly and may be better described as different scenarios.

One implication of uncertainty is that as new information is revealed estimates of LRMC may need to vary significantly. For example, consider a demand forecast which includes a

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24 That is, a value that is weighted by the likelihood of it occurring.
scenario with 50 per cent probability that there will be significant growth in development that will drive the need for investment and therefore contribute to a higher expected LRMC. If pricing is based on expected LRMC then the price will fall if this new development does not eventuate and rise if it does. Whichever scenario eventuates, the true LRMC will differ from expected LRMC.

### 3.2.4 Increasing long run costs of supply

A common feature of utility industries is that due to locational constraints the LRAC of supply is increasing; that is, for example, each successive increase in capacity is more expensive than the last.\(^{25}\)

An implication of increasing long run costs of supply is that using a longer time period for determining LRMC will yield a greater value of LRMC.

### 3.2.5 LRMC pricing and demand

There is circularity in calculating LRMC and using it as a basis for pricing. Estimates of LRMC will depend on forecasts of future demand, which in turn will depend on the price that is set with regard to LRMC. Therefore, it is necessary to consider the effects of price when estimating LRMC.

In some situations a price set to the LRMC would reduce demand sufficiently to negate the need for the augmentation. In such situations, it would be efficient to set the price at the level that just reduces demand sufficiently to negate the need for the augmentation. In effect, the LRMC in such situations is the LRMC of the demand reduction required.

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\(^{25}\) LRMC may also be increasing over time due to increasing marginal operating costs to meet additional demand.
4. Pricing based on LRMC

The discussion in the previous section identifies trade-offs in setting prices with regard to LRMC relating to efficiency in the short-run and long-run. This section discusses considerations in evaluating these trade-offs.

4.1 Considerations

4.1.1 Efficiency

The primary purpose of setting price to LRMC (or other measure of marginal cost) is to provide a signal to consumers and suppliers for efficient use. The preferred time period over which to calculate marginal cost depends significantly on the extent to which consumers respond to price signals.

The responsiveness to price may be influenced by a number of factors relevant to how usage prices may be set. These include the following:

- The nature of shocks to supply and demand.
  Shocks to the demand and supply balance are arguably more likely to be negative (e.g. drought or an energy supply disaster) than positive and therefore large adjustments are more likely to involve increases rather than decreases to LRMC.

- The direction of price change.
  There is empirical evidence that the responsiveness to price changes is asymmetric; consumers are more sensitive to price increases than price decreases.\(^{26}\)

- The speed of price change.
  More rapid price changes attract more attention and may receive a greater response. Therefore, for example, a stable price followed by a sudden price increase may obtain a greater response than a gradual increase in prices over time.

- How prices are communicated.
  The responsiveness to price signals will depend in part on how prices and changes in prices are communicated. The price signal to consumers may be poorly communicated because:
    - complex price structures such as inclining block tariffs (IBTs) make it difficult for consumers to understand what marginal price they pay
    - prices are generally only set for a regulatory period and the project price path is generally not clearly communicated.

\(^{26}\) For example, Gately and Huntington (2002) found that demand in OECD countries responds much more to increases in oil prices than to decreases. More recently Sentenac-Chemin (2012) found empirical evidence that for gasoline prices households are more sensitive to price increases than price decreases.
Price strategies might be modified in light of these considerations. For example, it may be preferable to set prices low and raise prices temporarily in response to supply and demand shocks rather begin with high prices. Similarly it may be preferable to set

4.1.2 Equity and consumer acceptance of usage prices

Variations in usage prices (over time and across different locations) have implications for consumer bills and therefore perceptions of equity, consumer acceptance of price changes and the revenue of the utilities. For example, the impact on consumer bills is often provided as a reason against short-term pricing variations such as changing water usage prices during a drought.

The impact of usage price variations on consumer bills (and potentially perceptions of equity) depends critically on how non-usage charges are set and modified. To achieve revenue stability, a decrease (increase) in usage prices necessitates that other prices must rise (fall). It is therefore of interest to know whether pricing policies, which include usage and non-usage charges, can be developed that are perceived as equitable and minimise the impact on consumer bills from usage pricing variation over time and across locations.

Such policies appear possible. Variations in usage prices might be offset by variations in other charges in such a way that the bill of a consumer with a particular usage does not change with changes in aggregate demand. A useful framework is to think of consumers having an entitlement to a particular level of usage. For example if a consumer entitlement was 200 kL of water per year then the bill of a consumer who uses 190 kL would decrease as usage prices increase and vice-versa.

4.2 Pricing options

The choices in setting pricing with regard to LRMC can be categorised into how a) should prices vary over time, and b) how should prices be cater for uncertainty.

4.2.1 How should prices vary over time

With regard to variation over time there are two possible extremes to pricing based on LRMC.

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27 Assuming the demand response is price inelastic. If demand was elastic then the revenue collected from usage charges would increase following a price decrease (due to the greater quantities consumed).

28 The 'entitlement' based framework is robust to increases in aggregate demand. Increases in demand may come from new consumers and/or from increases in average use by existing consumers. An increase in average use changes the usage price but does not affect the entitlement of existing consumers and thus does not affect the total bill of consumers who use just their entitlement. To accommodate an increase in the number of consumers (which necessitates a decrease in the average entitlement per consumer), existing consumers can be compensated from charges levied on new consumer; which for perceptions of fairness may be best captured via charges on developers. Similarly an entitlement based framework is also useful to considering changes in supply. A permanent negative shock to supply (e.g. a reduction in supply from climate change) necessitates a reduction in entitlements; whereas the development of new supply increases the available entitlement.
At one extreme, pricing could follow the saw-tooth pattern shown in Figure 4 above using the Turvey or AIC methods.

At the other extreme an option is to use average of the LRMC; that is to set a price that smooths over the saw-tooth pattern. If the cost of additional capacity (and rate of demand growth) is constant then the average of the Turvey LRMC will be the unit cost of the augmentation. Thus a simplified LRMC pricing rule may be to simply set LRMC to the annuitized unit cost of the next augmentation. However, while this is simple and leads to a stable price, the approach accentuates many of the issues with LRMC including the issues discussed above relating to sending an inefficient signal in the short-run, particularly when the next investment required is in the distant future.

When the next investment required is in the distant future, an alternative, that is in-between the two extremes, might be preferable whereby prices are initially stable and low and then when the required investment is sufficiently close rise to a new level that reflects the revised LRMC. Such an approach might be desirable to maximise stability and communication of the price signal.

Researchers and practitioners on the subject of LRMC have generally either avoided discussing the issue or expressed a preference without detailed exposition. For example

- In his 1976 paper (Turvey 1976, page 168) Turvey states that ‘the analysis of marginal costs by year does not necessarily imply that charges which reflect marginal cost should vary from year to year (apart, of course, from adjustments to inflation)’.
- London Economics (1997, page 19) argued for smoothing but provided very light justification. They state that ‘prices based on LRIC [AIC] would lead to unacceptable variation in prices from year to year’ and argued that a ‘Consequently, a pricing rule is needed which would on average allow companies to recover future costs of meeting an increase in demand without these fluctuations in prices.’

### 4.2.2 Pricing given uncertainty

The estimate of LRMC may depend significantly on forecasts of supply and demand that are uncertain. A common approach to estimating values under uncertainty is use the expected value; that is the probability-weighted average value.

However alternatives may be considered. In particular, it may be preferable to set the price based a low-price scenario and then if the high-price scenario develops apply an adjustment. This approach may be preferable on the basis that:

- the nature of supply shocks means that large price increases are more likely than large price falls, and
- consumers are likely to be more responsive in the short-term to price rises than falls (see footnote 26 above).

### 4.3 Conclusion

Applying LRMC pricing involves balancing competing objectives associated with:

- the efficiency of the price-signal in the short and the long-run
- the stability of prices, and
• administrative simplicity.

The key choices relate to the extent to which prices are smoothed over time and how uncertainty in forecasts is reflected.

Perceptions of equity (i.e. fairness) are also a consideration. Given the potential for prices set with regard to LRMC to change over time it is important to have a clear framework for other charges to manage the impact of customer bills.

A summary of the relative benefits of the main different marginal cost methods is shown in Table 1, ordered by the degree to which they deviate from SRMC. There is no single pricing rule based on LRMC that is clearly superior. Broadly, the more that consumers and suppliers are price responsive the more the greater emphasis that should be place on short run costs.

**Table 1: Summary of cost measures (in order of variability)**

<table>
<thead>
<tr>
<th>Cost measure</th>
<th>Pros / cons</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRMC</td>
<td>+ Provides an efficient price signal for short-run decisions</td>
</tr>
<tr>
<td></td>
<td>− High variability of prices associated capacity constraints and augmentations</td>
</tr>
<tr>
<td>SRMC with storage</td>
<td>+ Provides an efficient price signal for short and long-run decisions</td>
</tr>
<tr>
<td></td>
<td>+ Dampens some variability associated with SRMC but retains significant variability</td>
</tr>
<tr>
<td></td>
<td>− Only applicable when there is sufficient cheap storage</td>
</tr>
<tr>
<td>AIC LRMC</td>
<td>+ Significantly dampens (but does not remove) variability associated with SRMC</td>
</tr>
<tr>
<td></td>
<td>− Issues with LRMC pricing including sends inefficient price signal for short-term decisions and sensitive of results to length of period chosen</td>
</tr>
<tr>
<td></td>
<td>◦ Relative to Turvey method places additional weight on costs incurred in near future. This may be considered less accurate as a measure of LRMC but has an advantage when costs in the long-term are highly uncertain and/or greater emphasis is wanted on short run costs</td>
</tr>
<tr>
<td>Turvey LRMC</td>
<td>+ More accurately measure the cost of a marginal permanent change in demand</td>
</tr>
<tr>
<td></td>
<td>+ May further dampen variability when there are large capital investments</td>
</tr>
<tr>
<td></td>
<td>+ Easier to use to isolate the LRMC of a single cost driver</td>
</tr>
<tr>
<td></td>
<td>− Accentuates issues with LRMC pricing (e.g. inefficient signals in short-term)</td>
</tr>
<tr>
<td>Annuitized unit cost of next supply source</td>
<td>+ Approximates the average Turvey LRMC over time thereby removing additional variability</td>
</tr>
<tr>
<td></td>
<td>+ Simple to calculate</td>
</tr>
<tr>
<td></td>
<td>− Further accentuates issues with LRMC pricing.</td>
</tr>
</tbody>
</table>
References


Kahn AE. (1988), The economics of regulation: Principles and Institutions, Volume I.


Marsden Jacob Associates, (2004). Estimation of Long Run Marginal Cost (LRMC) - A report prepared by the Queensland Competition Authority - Final, 3 November 2004


Appendix 1 Algebraic representation of LRMC

Simple model

Set up
Assume that:

- we wish to estimate LRMC over a planning period from time $t = 0$ to $t = T$
- demand is growing by a constant amount of $q$ units per year
- the discount rate is $r$ and, for convenience, denote $d_t$ as the discount factor for period $t$; that is $d_t = (1 + r)^{-t}$
- supply is increased at least cost by investments with capital cost of $I_t$, which are infinitely lived (or can be replaced indefinitely at the same annuitized cost) and have an annuitized cost of $ml_t$, where $m$ is a parameter that reflects the cost of financing, depreciation and maintenance applicable for the investment
- marginal operating costs of $c_t$. For convenience let $H$ denote present value of future capital expenditure and $\sum = \sum^T_{t=0}$; thus $H = \sum d_t I_t$.

AIC method

First, consider the PV of costs associated with a single future investment in capacity that occurs at time $t$. This PV is the discounted sum of annuitized cost on the investment, which is $ml_t(d_t + d_{t+1} + \cdots + d_T) = w_t ml_t$; where $w_t = \sum^T_{j=t} d_j$.

Similarly, the PV of additional permanent demand beginning in period $t$ is $w_t q$ and the PV of all incremental demand is $\sum w_t q$.

The capital component of AIC can therefore be calculated as:

$$AIC_{LRCMC} = \sum^{T}_{t=0} \frac{w_t ml_t}{\sum w_t q}$$

Similarly, the PV of operating expenditure required to meet the level of demand that is reached at time $t$ is the discounted sum of repeating that marginal operating expenditure each year in the future, which is $w_t c_t q$.

The PV of the operating cost component of the AIC is therefore:

$$AIC_{LRMOC} = \sum^{T}_{t=0} \frac{w_t c_t}{\sum w_t}$$

Combining the two components:
\[ \text{AIC LRMC} = \sum_{t=0}^{T} \frac{w_t}{\sum w_t} \left( \frac{m l_t}{q} + c_t \right) \]

This can be thought of as the weighted average of per-unit costs \((m l_t/q + c_t)\) into the future.

**Turvey method**

\[ \text{Turvey LRMC} = \frac{\text{PV of costs to meet a permanent change in demand}}{\text{PV of permanent change in demand}}. \]

The Turvey method requires the selection of a marginal permanent change in demand.

For the purposes of comparison with the AIC method it is useful to consider the effect of a permanent reduction in demand of \(q\) units, thereby postponing all capital expenditure by a single period. In such case: 29

- the PV of a permanent change in demand is a reduction of \(\sum d_t q\),
- the PV of change in capital costs to meet the demand is the reduction cost of postponing the capital expenditure by a single period which is \(\sum_{t=0}^{T} d_t m l_t\) and therefore:

\[ \text{Turvey LRMCC} = \sum_{t=0}^{T} \frac{d_t}{\sum d_t} m l_t \]

- the PV of change in operating costs to meet the demand is the reduction cost of postponing the operating expenditure by a single period which is \(\sum_{t=0}^{T} d_t c_t q\)

\[ \text{Turvey LROMC} = \sum_{t=0}^{T} \frac{d_t}{\sum d_t} c_t \]

Combining the two components:

\[ \text{Turvey LRMC} = \sum_{t=0}^{T} \frac{d_t}{\sum d_t} \left( \frac{m l_t}{q} + c_t \right) \]

Similar to the AIC LRMC calculated above, this Turvey LRMC is the weighted average \((\sum d_t)\)

of future per-unit costs (being, \(m l_t/q + c_t\)).

**Comparing AIC and Turvey LRMC**

In the above example, the AIC and Turvey LRMC measures can be thought of weighted averages of future per-unit costs. Due to discounting both weights fall over time, however as demonstrated in Figure 5 below the AIC weights fall at a faster pace and therefore the AIC LRMC places greater weight on earlier costs.

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29 If a permanent increase in demand is used the formulas shown would change slightly to reflect costs are being brought forward.
Figure 5: Comparison of Turvey and AIC weights

Note: Graph constructed using a discount factor of 5% and planning period of 25 years.

The difference between the weighting depends on the discount rate and the time period used. The greater the discount rate and/or the greater the time period used the more that the weights will converge. Figure 6 below shows the effect of a greater discount rate.

Figure 6: Comparison of Turvey and AIC weights (15% discount rate)

The rate of decline of the weights can be calculated as follows. For the Turvey weights the ratio of two consecutive weights is \( \frac{d_{t+1}}{d_t} = \frac{1}{1+r} = d_1 \). For the AIC weights, note that \( w_{t+1} = \sum_t d_t = d_1 (\sum_t d_t) - d_T = d_1 w_t - d_T \) and therefore \( \frac{w_{t+1}}{w_t} = d_1 - \frac{d_T}{w_t} \).
Changing LRMC over time

How does LRMC change in between investments?

Assume that the next investment occurs in \(j\) periods (i.e. \(I_t = 0\) for \(t < j\)), the length of the planning period does not change (i.e. LRMC is always calculated over \(T+1\) periods) and there is no investment following the planning period (i.e. \(I_{T+1+t} = 0\) for \(t < j\)). Then in one additional period’s time the Turvey LRMC calculation will be identical in all respects with the exception that the costs will be brought forward by one period, which due to discounting will increase in PV terms by the discount rate. That is:

\[
\text{Turvey LRMC}_{t=1} = \sum_{t=1}^{T+1} \frac{mI_t}{\sum d_t / q}
\]

Since \(I_0 = 0\) and \(I_{T+1} = 0\) and \(d_{t-1} = (1 + r) d_t\)

\[
\text{Turvey LRMC}_{t=1} = (1 + r) \sum_{t=0}^{T} \frac{d_t}{\sum d_t} \frac{mI_t}{q}
\]

\[
= (1 + r) \times \text{Turvey LRMC}_{t=0}
\]

Therefore for time periods \(t < j\) the Turvey LRMC will grow at the rate of \(1 + r\).

The average value of LRMC between investments

Assume that each investment has capacity \(k\) and unit cost \(u_t\); that is \(I_t = ku_t\). For ease of exposition assume \(q = 1\) and therefore the time between investments is \(k\). Assume the planning period begins immediately after an investment and lasts for \(T = nk\) periods. For convenience label \(D_t = \sum_{j=t}^{T+k-1} d_j\).

Assume initially that \(n = 1\) and therefore \(\sum d_t = D_0\).

Consider the \(k\) periods from immediately after an investment to immediately before the next investment. The LRMC equals \(\frac{d_k m_l_k}{\sum d_t}\) and then escalates at \((1 + r)\).

The average of Turvey LRMC over this period is therefore, equal to:

\[
\frac{d_k m_l_k (1 + (1 + r) \ldots (1 + r)^{k-1})}{\sum d_t / k} = \frac{d_k m_l_k D_0 / d_k}{\sum d_t / k}
\]

\[
\frac{m d_k k u_k D_0}{D_0 / k d_k} = m u_k
\]

That is, under these assumptions the average value of the Turvey LRMC over the period between investments is simply the annuitized unit cost of the upcoming investment.